

1434

Faculty of Science  
Mathematics DepartmentGeneral Topology  
Final ExamDate: 8 /7 /1434  
Time:8- 10

Name:

Co. No.

Serial number

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Answer the following:

(QI) Mark true or false and justify your answer.

1. Let  $(X, \tau)$  be a topological space and  $A \subset X$ . Then  $A^c \subset ext(A)$ . 0.52.  $x \in A'$  iff  $x \in \overline{A - \{x\}}$  1.53. Let  $A$  be any subset of a topological space  $X$ , then  $b(A) = \varnothing$  if and only if  $A$  is both open and closed. 24. Every finite  $T_1$ -space  $X$  is a discrete space. 1.55. If  $A \subset B$ , then  $\overline{B} \subset \overline{A}$ . 1.5

6. The identity map  $i : (X, \tau_1) \rightarrow (X, \tau_2)$  is continuous if and only if  $\tau_1 \subset \tau_2$ . 2

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7. The discrete space  $X$  is separable if and only if it is countable. 2

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8. Let  $A$  be a nonempty set of a metric space  $X$ . Then  $x \in A$  iff  $d(x, A) = 0$ . 2

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9. Any subspace of a second countable space is second countable. 2

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10. Every indiscrete space is disconnected. 1

(QII) Prove the following:

1. Let  $f: X \rightarrow Y$  be a function from the topological space  $(X, \tau_X)$  into the topological space  $(Y, \tau_Y)$ . Prove the following are equivalent: 3

- a. for each closed set  $B$  in  $Y$ ,  $f^{-1}(B)$  is closed in  $X$ ,
- b. for each subset  $B \subset Y$ ,  $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$

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2. Every compact Hausdorff space is normal. 5

3. Let  $C$  be a connected subset of the space  $X$ . Then

a. every set  $B$  such that  $C \subset B \subset \bar{C}$  is also connected, 3

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b.  $\bar{B}$  is connected 1

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4. Let  $(X, d)$  be a metric space. Then the set of all open balls in  $X$  is a base for a topology on  $X$ . 4

5. Compactness is a topological property. 4

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6. Let  $A$  be a closed subset of a  $T_4$ -space. Show that  $A$  with the relative topology is also  $T_4$ -space. 4