King Abdul Aziz University **Math 312 CAR** **Second Semester 1433/ 1434**

Faculty of Science **Real Analysis II** **Date: 16 /7 /1434**

Mathematics Departement **Final Exam** **Time: 10.30 - 12.30**

Name: **Co. No.** **Serial number**

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Answer the following:

1. State and prove Banach Fixed point Theorem. [6]

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2. Suppose f is continuous real function on a compact metric space S and $M = \sup_{p \in S} f(p),\ m = \inf_{p \in S} f(p)$.

Prove that $M, m \in R_f$. [4]

3. Let M be a nonempty subset of a metric space X. Prove that $x \in \overline{M}$ iff there is a sequence (x_n) in M such that $x_n \to x$. $\boxed{5}$

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4. Prove that f is Reimann integrable on $[a, b]$ iff for every $\epsilon > 0$ there exists a partition P such that $U(P, f) - L(P, f) < \epsilon$. $\boxed{5}$

5. Suppose E is an open set in $\mathbb{R}^n$, f maps E into $\mathbb{R}^m$, f is differentiable at $x_0 \in E$, g maps an open set containing $f(E)$ into $\mathbb{R}^k$, g is differntiable at $f(x_0)$, then the mapping F of E into $\mathbb{R}^k$ defined by $F(x) = g(f(x))$ is differntiable at x_0 and $F'(x_0) = g'(f(x_0))f'(x_0)$. 6

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6. Suppose that $X = (x_n)$ and $Y = (y_n)$ are positive real sequences. If $\lim_{n\to\infty} \frac{x_n}{y_n} \neq 0$, then prove that $\sum x_n$ is convergent iff $\sum y_n$ is convergent. 3

7. Suppose $f_n \to f$ uniformly on a set E in a metric space, let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n, n = 1, 2, 3, \ldots,$ then $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$. 3

a. ______________________________

b. under what condition f is continuous? prove? 2

8. Let E be an open set in $\mathbb{R}^n$, suppose f is a C'- mapping of an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^n$, $f'(a)$ is invertible for some $a \in E$ and $b = f(a)$ then $\exists$ open sets U and V containing a and b respectively such that f is one to one on U and $f(U) = V$. 6